Cambridge International Examinations
Cambridge Ordinary Level

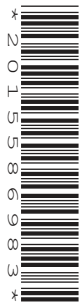| CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

ADDITIONAL MATHEMATICS **4037/11**

Paper 1 **May/June 2016**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\csc^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Find the value of k for which the curve $y = 2x^2 - 3x + k$

    (i) passes through the point $(4, -7)$,   [1]

    (ii) meets the x-axis at one point only.   [2]

2   **(a)** Solve the equation $16^{3x-1} = 8^{x+2}$.   [3]

    (b) Given that $\dfrac{\left(a^{\frac{1}{3}}b^{-\frac{1}{2}}\right)^{3}}{a^{-\frac{2}{3}}b^{\frac{1}{2}}} = a^{p}b^{q}$, find the value of each of the constants p and q.   [2]

3 Find the equation of the normal to the curve $y = \ln(2x^2 - 7)$ at the point where the curve crosses the positive x-axis. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

4   **(a)** Given the matrices $\mathbf{A} = \begin{pmatrix} -1 & 2 \\ 3 & 0 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$, find $\mathbf{A}^2 - 2\mathbf{B}$.   [3]

(b) Using a matrix method, solve the equations

$$4x + y = 1,$$
$$10x + 3y = 1.$$ [4]

       [Turn over

5   **Do not use a calculator in this question.**

   (i)   Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\mathrm{e}^{4x}}{4} - x\,\mathrm{e}^{4x}\right) = px\,\mathrm{e}^{4x}$, where p is an integer to be found.   [4]

   (ii)   Hence find the exact value of $\displaystyle\int_{0}^{\ln 2} x\,\mathrm{e}^{4x}\,\mathrm{d}x$, giving your answer in the form $a\ln 2 + \dfrac{b}{c}$, where a, b and c are integers to be found.   [4]

6 The function f is defined by $\quad f(x) = 2 - \sqrt{x+5} \quad$ for $-5 \leqslant x < 0$.

(i) Write down the range of f. [2]

(ii) Find $f^{-1}(x)$ and state its domain and range. [4]

The function g is defined by $\quad g(x) = \dfrac{4}{x} \quad$ for $-5 \leqslant x < -1$.

(iii) Solve $fg(x) = 0$. [3]

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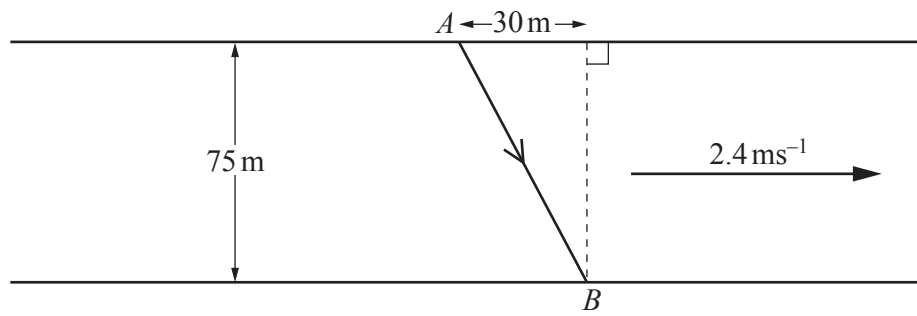
The diagram shows a river with parallel banks. The river is 75 m wide and is flowing with a speed of 2.4 ms^{-1}. A speedboat travels in a straight line from a point A on one bank to a point B on the opposite bank, 30 m downstream from A. The speedboat can travel at a speed of 4.5 ms^{-1} in still water.

(i) Find the angle to the bank and the direction in which the speedboat is steered. [4]

(ii) Find the time the speedboat takes to travel from *A* to *B*. [4]

8   **Solutions to this question by accurate drawing will not be accepted.**

Three points have coordinates $A(-8, 6)$, $B(4, 2)$ and $C(-1, 7)$. The line through C perpendicular to AB intersects AB at the point P.

(i)   Find the equation of the line AB.   [2]

(ii)   Find the equation of the line CP.   [2]

(iii)   Show that P is the midpoint of AB.   [3]

(iv) Calculate the length of *CP*. [1]

(v) Hence find the area of the triangle *ABC*. [2]

[Turn over

9   **(i)**   Show that $2\cos x \cot x + 1 = \cot x + 2\cos x$ can be written in the form
 $(a\cos x - b)(\cos x - \sin x) = 0$,   where a and b are constants to be found.   [4]

  (ii)   Hence, or otherwise, solve $2\cos x \cot x + 1 = \cot x + 2\cos x$ for $0 < x < \pi$.   [3]

10 (i) Given that $f(x) = 4x^3 + kx + p$ is exactly divisible by $x + 2$ and $f'(x)$ is exactly divisible by $2x - 1$, find the value of k and of p. [4]

(ii) Using the values of k and p found in part **(i)**, show that $f(x) = (x + 2)(ax^2 + bx + c)$, where a, b and c are integers to be found. [2]

(iii) Hence show that $f(x) = 0$ has only one solution and state this solution. [2]

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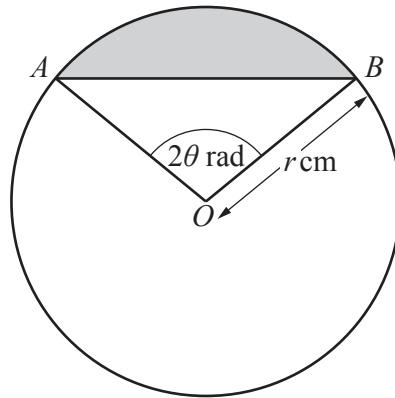
The diagram shows a circle, centre O, radius r cm. The points A and B lie on the circle such that angle $AOB = 2\theta$ radians.

(i) Find, in terms of r and θ, an expression for the length of the chord AB. [1]

(ii) Given that the perimeter of the shaded region is 20 cm, show that $r = \dfrac{10}{\theta + \sin\theta}$. [2]

(iii) Given that r and θ can vary, find the value of $\dfrac{\mathrm{d}r}{\mathrm{d}\theta}$ when $\theta = \dfrac{\pi}{6}$. [4]

(iv) Given that r is increasing at the rate of $15\,\mathrm{cm\,s^{-1}}$, find the corresponding rate of change of θ when $\theta = \dfrac{\pi}{6}$. [3]

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